$$\triangle=\begin{vmatrix} -\frac{yz}{x^2}, & \frac{z}{x}, & \frac{y}{x} \\ \frac{z}{y}, & -\frac{xz}{y^2}, & \frac{x}{y} \\ \frac{y}{z}, & \frac{x}{z}, & -\frac{xy}{z^2} \end{vmatrix}=xyz\begin{vmatrix} -\frac{1}{x}, & \frac{1}{y}, & \frac{1}{z} \\ \frac{1}{x}, & -\frac{1}{y}, & \frac{1}{z} \\ \frac{1}{x}, & \frac{1}{y}, & -\frac{1}{z} \end{vmatrix}=xyz\begin{vmatrix} 0, & 0, & \frac{2}{z} \\ \frac{2}{x}, & 0, & 0 \\ \frac{1}{x}, & \frac{1}{y}, & -\frac{1}{z} \end{vmatrix}=4.$$

The value announced, $4xyz$, is therefore erroneous.

Solved similarly by J. Scheffer and W. J. Greenstreet.

MECHANICS.

356. Proposed by the late G. B. M. ZERR, Ph. D.

A cantilever beam length a is loaded with c pounds per running foot at its fixed end and increases uniformly to b pounds per running foot at its free end. Find the deflection at the free end due to this load.

Solution by FRANCIS RUST, E. E., Pittsburg, Pa.

The problem is solved by determining the elastic curve of the beam's axis from its second differential equation

$$y''=\frac{M}{IE},$$

as derived from Hooke's law, to be found in all text books. M is the bending moment in the point, abscissa$=x$; I is the moment of inertia of the beam's cross-section, and E is the modulus of elasticity of its material.

M is to be determined by the arrangement of the load with mathematical certainty. Taking the point, abscissa$=x$, for origin,

$$M=\int_0^{a-x} q\xi\, d\xi,$$

q, the load per lineal foot, to be expressed as a function of the variable of integration ξ. $q=c+px$, and $c+pa=b$. Consequently,

$$p=\frac{b-c}{a},$$

and q as a function of ξ is

$$q=c+\frac{b-c}{a}.(x+\xi)=\frac{ac+(b-c)x}{a}+\frac{b-c}{a}\xi, \text{ and}$$

$$M=\frac{ac+(b-c)x}{a}\int_0^{a-x}\xi\,d\,\xi+\frac{b-c}{a}\int_0^{a-x}\xi^2 d\,\xi=\frac{ac+(b-c)x}{a}.\frac{(a-x)^2}{2}$$

$$+\frac{b-c}{a}.\frac{(a-x)^3}{3}=\frac{a^2}{6}\left(1-\frac{x}{a}\right)^2\left((2b+c)+(b-c)\frac{x}{a}\right)$$

which, properly simplified, gives us

$$M=\frac{a^2}{6}\left[(2b+c)-3(b+c)\frac{x}{a}+3c\left(\frac{x}{a}\right)^2+(b-c)\left(\frac{x}{a}\right)^3\right].$$

Assuming now a beam of uniform cross-section and homogeneous material, and introducing the new variable, $x/a=u$, we have $dx=adu$.

The integration of $y''=M/IE$ may be performed without any difficulty. It gives the equation of the elastic curve,

$$y=\frac{a^4}{6EI}.\left[\frac{2b+c}{2}\left(\frac{x}{a}\right)^2-\frac{b+c}{2}\left(\frac{x}{a}\right)^3+\frac{c}{4}\left(\frac{x}{a}\right)^4+\frac{b+c}{20}\left(\frac{x}{a}\right)^5\right]$$

from which is derived for $x=a$,

$$y_a=f=\frac{a^4(11b+4c)}{120EI}.$$

Remark. This result is as simple as it is wrong.

In my memoir *Der Fehler in Hooke's Gesetz,* published in Oesten, Wochensch. f. d. aff. Bandienst, year 1900, p. 252, ff., I have deduced mathematically and proved by experiment, that Hooke's law *cannot be applied on the beginning of deformation.*

The locus of any elastic curve, deduced from Hooke's law, *p. e.* our equation above, is a continuous curve, whilst the experiment shows, that a cantilever (spring under its own weight, for instance) will remain *unbent for a certain length at its free end.*

The investigation into these conditions reveals the fact, that any material may be submitted to a certain stress, which must be overcome before deformation takes place. Such minimal stress may be called *the resistance of inertia* (Tragheits widerstand). Denoting it by α, the true expression of the law of elastic deformation is

$$\triangle l:l=\sqrt{(\sigma^2+\alpha^2)}:E.$$